Reaching the Mountaintop: Addressing the Common Core Standards in Mathematics for Students with Mathematics Difficulties

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The Common Core State Standards provide teachers with a framework of necessary mathematics skills across grades K-12, which vary considerably from previous mathematics standards. In this article, we discuss concerns about the implications of the Common Core for students with mathematics difficulties (MD), given that students with MD, by definition, struggle with mathematical skills. We suggest that instruction centered on the Common Core will be challenging and may lead to problematic outcomes for this population. We propose that working on foundational skills related to the Common Core standards is a necessary component of mathematics instruction for students with MD, and we provide teachers with a framework for working on foundational skills concurrent with the Common Core standards. We caution, however, that implementation of the Common Core is in its infancy, and the implications of the Common Core for students with MD need to be monitored carefully.

The Common Core State Standards Initiative provides teachers with essential components of mathematics instruction across grades K-12 (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). In this article, we refer to the set of standards as the Common Core. As of fall 2012, the Common Core has been formally adopted by 45 states. At grades K-8, the Common Core identifies the content and practices teachers should address at each grade. For students in grades 9–12, the Common Core classifies the relevant content into six conceptual categories (i.e., Number and Quantity, Algebra, Functions, Modeling, Geometry, and Statistics and Probability). High-school teachers may teach the standards from the conceptual categories across several grade levels or courses.

The Common Core is a collection of standards for mathematical practice and mathematical content. The eight standards for mathematical practice, listed in Table 1, highlight important ways in which teachers should teach mathematics. Teachers are encouraged to provide opportunities to implement the practice standards throughout elementary, middle, and high school alongside the mathematical content standards (Russell, 2012). By contrast, the standards for mathematical content are a collection of hundreds of mathematics standards. The content standards provide teachers with an outline of what should be taught. In this article, we focus on the standards for mathematical content. At each grade level, the Common Core is divided into domain areas (e.g., Counting and Cardinality, Measurement and Data, and Functions). See Figure 1 for a list of domains by grade level. Each domain is broken into clusters or related standards, and the standards fall into any of the 1–5 clusters per domain. See Figure 2 for an example from fifth grade in the Number and Operations – Fractions domain area. The cluster is “Use equivalent fractions as a strategy to add and subtract fractions.” Standards #1 and #2 relate to the cluster and domain.

The Common Core was designed as a blueprint for mathematics instruction in the general education classroom. However, because many students, either diagnosed with a learning disability in mathematics or demonstrating below grade-level mathematics performance, receive their mathematics instruction in the general education classroom, questions arise about the best way to provide intervention for these students while simultaneously addressing the Common Core. In this article, we address these questions by discussing how the Common Core relates to mathematics instruction for students with mathematics difficulties (MD).

WHAT THE STANDARDS ADDRESS

The Common Core State Standards in Mathematics document provides an in-depth explanation of what should be taught at each grade level. Although we summarize the domain areas, clusters, and standards, we suggest teachers download and become familiar with the original document.
TABLE 1
Common Core Standards for Mathematical Practice

<table>
<thead>
<tr>
<th>Number</th>
<th>Standard</th>
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<tbody>
<tr>
<td>1</td>
<td>Make sense of problems and persevere in solving them.</td>
</tr>
<tr>
<td>2</td>
<td>Reason abstractly and quantitatively.</td>
</tr>
<tr>
<td>3</td>
<td>Construct viable arguments and critique the reasoning of others.</td>
</tr>
<tr>
<td>4</td>
<td>Model with mathematics.</td>
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<tr>
<td>5</td>
<td>Use appropriate tools strategically.</td>
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<tr>
<td>6</td>
<td>Attend to precision.</td>
</tr>
<tr>
<td>7</td>
<td>Look for and make use of structure.</td>
</tr>
<tr>
<td>8</td>
<td>Look for and express regularity in repeated reasoning.</td>
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(www.corestandards.org/the-standards/mathematics). In the elementary grades (K-5), the goal of the Common Core is to provide students with a strong background on the fundamentals of mathematics. At Kindergarten, teachers focus on establishing strong counting skills with 0–100, helping students compare quantities, and working on the basic principles of addition and subtraction. In first and second grade, the Common Core emphasizes instruction on addition and subtraction and place value. Teaching about multiplication and division starts at third grade and continues into fourth grade. At grades 3, 4, and 5, place value and fractions constitute the major focus. By the end of fifth grade, the Common Core expects students to multiply and divide fractions. Across grades K-5, the Common Core emphasizes learning about measurement and geometry. For the measurement and data domain, this ranges from measuring length and weight to solving story problems about elapsed time. In geometry, the focus is on identifying shapes (in Kindergarten) to measuring angles and working with coordinate planes (in grades 4–5).

Geometry continues as a domain area in the middle-school grades (6–8). The geometry standards include, for example, calculating area (in sixth grade) to learning when to apply the Pythagorean theorem (in eighth grade). At grades 6–7, the Common Core emphasizes learning about ratios and proportions, and students across grades 6–8 continue to learn about whole and rational numbers, while working on statistics and probability. The biggest shift from the elementary to middle grades is in terms of pre-algebra skills. Beginning at sixth grade, the Common Core outlines standards on algebraic expressions and equations, and by the end of eighth grade, students should demonstrate competency with functions.

Number and Quantity, Algebra, Functions, Modeling, Geometry, and Statistics and Probability are the six conceptual categories for students in grades 9–12. The Number and Quantity standards address complex calculations as well as vectors and matrices. The Common Core Algebra category highlights solving equations and inequalities as well...
as performing complex calculations with polynomials. Linear, quadratic, exponential, and trigonometric functions are stressed in the Functions category. Geometry standards focus on synthetic and analytic aspects of Euclidean geometry, whereas the Statistics and Probability standards focus on interpreting data and calculating expected outcomes. The Modeling category does not list specific standards but emphasizes students making appropriate choices when presented with a problem and being able to formulate a model for a problem and compute, interpret, and validate an answer for the problem.

DIFFERENCES AND COMMONALITIES WITH CURRENT STATE STANDARDS AND THE NCTM STANDARDS

Porter, McMaken, Hwang, and Yang (2011) determined that the overlap between Common Core standards and state standards was only 20–35%. The Common Core differed in that its expectations emphasize higher-level thinking and conceptual understanding over memorization and procedures (Porter et al., 2011). Adoption of the Common Core therefore entails considerable change for schools in terms of teaching practices, professional development, and adoption of new curricula (Cobb & Jackson, 2011; Ediger, 2011; Lee, 2011).

Many states currently follow principles and standards outlined in Principles and Standards for School Mathematics (2000) by the National Council of Teachers of Mathematics for teaching mathematics from pre-Kindergarten through 12th grade. The six principles (equity, curriculum, teaching, learning, assessment, and technology) are the basic guidelines for mathematics instruction. The 10 standards describe what students should learn in mathematics in four grade bands: PreK-2, 3–5, 6–8, and 9–12. These NCTM standards comprise five content standards (Numbers and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability) and five process standards (Problem Solving, Reasoning and Proof, Communication, Connections, and Representation). These standards, especially the content standards, spurred on an era of “reform” mathematics, and many textbooks and assessments were written to align with the NCTM standards (Larson, 2012).

Beyond emphasizing higher-level thinking and conceptual understanding over memorization and procedures (Porter et al., 2011), similarities and differences between the NCTM standards and the Common Core include the following. The Common Core outlines highly specific standards to be taught at each grade level, whereas the NCTM Standards are described in grade bands and do not enumerate as many skills. For example, the fourth-grade Common Core includes 14 standards related to the learning of fractions. The NCTM Standards, by contrast, list four expectations (under the Numbers and Operations standards) for students in grades 3–5. Both sets of standards, however, cover the same umbrella skills of numbers, operations, algebra, measurement, geometry, and data analysis. What both sets of these standards have in common is their influence on mathematics instruction in the U.S. The NCTM Standards drove development and rewriting of textbooks and assessments for the last 20 years. Now the Common Core is likely to have the same impact (Larson, 2012). In fact, although the Common Core is relatively new, most states quickly adopted these standards, and now school districts are rapidly enhancing their curriculum guidelines (and textbook and assessment purchases) to be in alignment.

STUDENTS WITH MATHEMATICS DIFFICULTY

The question is, how do students with learning disabilities participate in mathematics instruction designed to address the Common Core? It is estimated 3–6% of school-age students struggle with a mathematics learning disability (Shalev, Auerbach, Manor, & Gross-Tsur, 2000), and their struggles typically persist throughout their formal schooling and beyond. That is, students who demonstrate lower mathematics performance in kindergarten make smaller gains in mathematics during elementary school (Jordan, Kaplan, Ramineni, & Locuniak, 2009; Judge & Watson, 2011), and 95% of students identified with a mathematics learning disability before fifth grade continue to struggle with mathematics in high school (Shalev, Manor, & Gross-Tsur, 2005). Students with a mathematics learning disability typically experience difficulty with mathematics into adulthood (Butterworth, Varma, & Laurillard, 2011). Many more students struggle with low mathematics performance without a formal diagnosis of a mathematics learning disability MD, which is often referred to as MD. In this section, we discuss the difficulties of students with either a mathematics disability or difficulty and instruction for these students in the era of the Common Core. We use the acronym MD to refer to students with MD, with and without a formal diagnosis of mathematics learning disability.

AREAS OF DIFFICULTY

Students with MD can exhibit difficulty in one or many areas of mathematics. Although the nature of MD varies by student, several topic areas represent core challenges. One of the earliest areas of difficulty is counting and magnitude. Students with MD may struggle with counting skills (Geary, Hamson, & Hoard, 2000). Students frequently miscount or double-count items and often utilize inefficient counting strategies (Geary, Hoard, & Hamson, 1999). Students with MD also commonly experience unusual difficulty with cardinality (i.e., telling how many items are represented in a set; Butterworth, 2010). Related to counting, these students often struggle with telling time (Burny, Valcke, & Desoete, 2012), experiencing the greatest challenges with telling time to 5- or 1-minute intervals. Such difficulty is attributed to inefficient counting strategies. Perhaps a key deficit that underlies long-term mathematics success occurs with students’ appreciation of magnitude. Butterworth et al. (2011), for example, demonstrated that students with MD struggle with comparing numbers more than students without MD. This deficit is often manifested differentially when Arabic numerals are used to represent quantities, rather than when students have
to compare magnitudes represents in object arrays (De Smedt & Gilmore, 2011; Mazzocco, Feigenson, & Halberda, 2011; Rousselle & Noël, 2007).

Another challenge, thought to represent a signature deficit for students with mathematics learning disabilities, concerns fluency with and conceptual understanding of basic number combinations (i.e., simple 1-digit addition, subtraction, multiplication, and division problems; Andersson, 2008; Geary, Hoard, & Bailey, 2012; Mabbott & Bisanz, 2008). Some researchers hypothesize that students may have deficits with semantic memory, which causes difficulty not only with representing quantities with their Arabic symbols but also with automatic retrieval of simple arithmetic problems (Baroody, Bajwa, & Eiland, 2009; Geary, 2004). Mathematics intervention should be designed to ensure that students with MD are fluent with the associations between Arabic numerals and the quantities they represent, as well as with the 100 addition, 100 subtraction, 100 multiplication, and 90 division number combinations. These are essential foundations for success with other mathematics skills such as computation or solving equations (Carr & Alexeev, 2011). When solving 149 + 387, having to stop and rely on procedural counting strategies to answer 9 + 7 challenges working memory resources in ways that automatic retrieval does not. This is problematic because research indicates that many students with MD have limitations in working memory (Passolunghi & Cornoldi, 2008; Swanson & Beebe-Frankenberger, 2004).

Because intervention does not always ensure this foundational competence, it is not surprising that students with MD typically experience difficulty with solving multi-digit computation problems, such as 149 + 387, that require manipulation of numbers related to place value (Andersson, 2008; Chong & Siegel, 2008). Difficulty with such procedural computation exists at both elementary and secondary levels (Calhoon, Emerson, Flores, & Houchins, 2007). Researchers suggest students with MD have visual or spatial difficulty that interferes with understanding place value (Swanson & Jerman, 2006).

Problem solving is another area of challenge for students with MD (Fuchs, Seethaler, et al., 2008; Reikerås, 2009), an area that is exacerbated and often characterized not only by the kinds of number processing difficulties already discussed, but also by language comprehension deficits (e.g., Compton, Fuchs, Fuchs, Lambert, & Hamlett, 2012; Fuchs, Fuchs, Stuebing, et al., 2008; Fuchs, Geary, et al., 2010). And other higher-level mathematics skills, such as rational numbers (Bailey, Hoard, Nugent, & Geary, in press; Mazzocco & Devlin, 2008), geometry (Cawley, Foley, & Hayes, 2009), algebra (Impecoven-Lind & Foegen, 2010), and proportions and ratios (Jitendra et al., 2009), can cause difficulty for students with MD.

In addition to all these mathematics areas, reading difficulties may intersect with mathematics performance, and these students tend to experience differentially poor performance with word problems (Cirino, Fuchs, Tolar, & Powell, 2011; Fletcher, 2005; Geary et al., 2000; Tressoldi, Rosati, & Lucangeli, 2007; Vukovic, 2012). This may be in part due to their reading difficulty, but it occurs even when word problems are read aloud. It is therefore likely that language comprehension deficits underlie the word-problem challenges students with concurrent difficulty in reading and mathematics experience (Fuchs, Fuchs, & Compton, in press).

EVIDENCE-BASED INTERVENTIONS

Over the last 30 years, an extensive research base has accrued indicating that students with MD require explicit, systematic instruction (Carnine, 1997; Doabler et al., 2011; Fuchs, Fuchs, Powell, et al., 2008; Gersten et al., 2009; Kroesbergen & Van Luit, 2003). Explicit instruction typically encompasses a step-by-step teacher demonstration for a specific type of problem along with teacher-guided and independent practice using the step-by-step procedure. Students should learn heuristics, mnemonics, or strategies to help solve certain types of problems (Gersten et al., 2009; Maccini, Mulcahy, & Wilson, 2007). Students should also receive mathematics instruction that emphasizes conceptual and procedural learning (Gersten, Jordan, & Flojo, 2005) and that provides visual representations to help students understand the key mathematical concepts (Gersten et al., 2009; Witzel, Mercer, & Miller, 2003). Practice across mathematics skills should be cumulative (Strickland & Maccini, 2010), and ongoing, systematic progress monitoring should be used to determine whether and, if so, when and how programs need to be adjusted to ensure adequate learning (Fuchs, Fuchs, Powell, et al., 2008). The National Mathematics Advisory Panel supports many of these strategies for teaching students with MD (2008).

Researchers have developed and demonstrated the efficacy of explicit interventions, based on these explicit instruction principles, for students with MD. For example, Bryant, Bryant, Gersten, Scannuccia, and Chavez (2008) provided small-group tutoring for second-grade students with MD in 15 min sessions that occurred 3–4 times a week for 18 weeks. The lessons focused on number concepts (e.g., counting, number identification, and comparing numbers), place value, and addition and subtraction number combinations. Students with MD who received tutoring demonstrated significantly higher posttest scores, but scores were still lower than students without MD. Focusing on exclusively on number combinations, Burns, Kanive, and DeGrande (2012) utilized a computer program to build knowledge with third- and fourth-grade students with MD. After using the program 3 times a week for 8–15 weeks, students with MD demonstrated significant gains on number combinations compared to students with MD who did not practice with the computer program. Fuchs, Seethaler, et al. (2008) focused on word problems with third-grade students with MD. Students received explicit schema-based instruction on three types of word problems. At posttest, students who received tutoring outperformed those in a control group. Similarly strong results occurred on word problems when schema-based instruction was contrasted to an active tutoring condition focused on number combinations (Fuchs et al., 2009). Witzel et al. (2003) compared an algebra intervention using a concrete-representational-abstract (CRA) sequence against traditional algebra instruction over an entire school year. Students with MD who worked on expressions, transformations, and other algebraic tasks with manipulatives (i.e., concrete) and
pictures (i.e., representational) outperformed students with MD who did not receive the CRA instructional sequence.

Also, it is important to note that early provision of such interventions can enhance long-term mathematics learning trajectories (Dowker, 2005). But in this vein, it is equally notable that MD may be more persistent than early reading difficulty (Fuchs, Fuchs, & Compton, 2012), therefore requiring more ongoing or intermittent intervention. This is because the mathematics curriculum introduces new topics at regular intervals. In fact, these curricular “twists and turns” sometimes create late-emerging MD. For example, fractions represents an area of emphasis in the Common Core at fourth grade. This topic, which is foundational to algebra and other higher-order forms of mathematics (Siegler et al., 2012), creates challenges for some students who have not experienced earlier mathematics learning difficulty. But the principles of whole and rational fractions differ in fundamental ways. All whole numbers can be represented by a single numeral, have unique successors, never decrease with multiplication, never increase with division, and so on. None of these properties, however, is true of fractions (Siegler & Pyke, in press). This illustrates the challenges associated with mathematics, which may be less salient in the domain of reading.

In any case, regardless of the type of difficulty or when students first manifest difficulty, students with MD require additional support or intervention at school. As sampled in the previous paragraphs, many researchers have demonstrated the efficacy and effectiveness of mathematics intervention programs for students with MD, using randomized control trials (e.g., Fuchs, Seethaler, et al., 2008; Jitendra et al., 2009; Tournaki, Bae, & Kerekes, 2008; Witzel et al., 2003). Now teachers must undertake the challenge of combining validated interventions with the Common Core standards.

**COMMON CORE AND STUDENTS WITH MD**

The mathematics curriculum in the 45 states that have already adopted the Common Core is (or soon will be) aligned with the Common Core. Because the Common Core is influencing many aspects of mathematics instruction in general education and because students with MD are required to have access to the general education curriculum (Russo, Osborne, & Borreca, 2005), intervention for students with MD must align with the Common Core. In a related way, as districts and schools in these states modify their curriculum to mirror the Common Core, textbooks and other curriculum materials for MD intervention must also be Common Core aligned (Wu, 2011). Moreover, the adoption of the Common Core will influence the high-stakes assessments schools administer such as the Program for the Assessment of Readiness for College and Careers (PARCC) and the Smarter Balanced Assessment Consortia (SBAC; Larson, 2012). Because students with MD must participate in those assessment programs (Yell, Katsiyannis, & Shriner 2006), it is important that these students receive instruction that supports performance on those assessments. All this indicates that teachers of students with MD cannot proceed with business-as-usual.

It is, however, impossible to ignore that students with MD have deficits that make accessing the grade-level-appropriate mathematics standards challenging. While some researchers question the validity of the Common Core standards (Tienken, 2011), most states have committed to implementation, and teachers must determine how to serve students with MD in the era of the Common Core. If students with MD are expected to participate in assessments and classroom instruction centered on the Common Core, teachers need to develop meaningful access points for students and focus on developing foundational skills to a targeted standard, even as they continue to work on the Common Core standards (Larson, 2012; Tienken, 2011). Identifying the critical foundational skills for a targeted Common Core standard is a necessary first step to create access to a targeted standard, for understanding how to provide effective instruction related to a targeted standard, and for creating an appropriate justification for working on foundational skills as a means for creating access.

**WORKING ON FOUNDATIONAL SKILLS AND THE COMMON CORE**

Foundational skills, such as knowledge of numbers, counting, number combinations, operations, and algorithms, are necessary to complete many mathematics problems (Sayeski & Paulsen, 2010). These foundational skills are often areas of substantial difficulty for students with MD (Andersson, 2008; Butterworth, 2010; Geary et al., 2012). For example, knowledge of numbers, fluency with number combinations, and understanding place value and regrouping are important foundational skills to successfully complete an algorithm for 167–88. For solving a word problem about tripling a baking recipe, critical foundational skills include multiplicative reasoning, understanding fraction concepts, knowing how to read and use relevant word-problem information, an ability to multiply fractions (or use successive addition), and an understanding of the relationship among whole numbers, mixed numbers, and fractions.

The foundational skills for a Common Core standard can vary from standard to standard, but many foundational skills will be revisited time and time again. Understanding the concept of addition, a foundational skill, may be important in standards related to number combinations (e.g., \( 8 + 4 \)), multi-digit computation (e.g., \( 7143 + 275 \)), algebra (e.g., \( 4 + 2x = x + 12 \)), geometry (e.g., calculate the perimeter of a triangle: \( a + b + c \)), measurement (e.g., adding \( 2 \frac{1}{3} \) hours to determine elapsed time), statistics (e.g., find the median price of 10 different loaves of bread), and word problems (e.g., Derek bought 2 bats, 4 baseballs, and 1 glove for baseball season. How many items did Derek buy?).

Many students with MD who struggle with early foundational skills will continue to struggle because the foundational skills are paramount to competence with higher-order math skills (Carr & Alexeev, 2011). For example, to calculate the area of a right triangle \( \left[ \frac{b \times h}{2} \right] \), students must understand what a triangle looks like, how to measure sides of a triangle, what numbers represent, the meaning of base and height, how to multiply, how to divide, and how area is represented by square units. But even each of these foundational skills has its own set of foundational skills. Before students...
can multiply or divide, they must understand addition and subtraction. If students do not have basic multiplication number combinations memorized (or at least know strategies for solving multiplication combinations), it is difficult to calculate \( 9 \times 7 \). For more complex multiplication, like \( 13 \times 42 \), students must know how to apply at least one algorithm (i.e., a step-by-step procedure) to find the product of the “base times height.” Additionally, when calculating \( 13 \times 42 \) using a partial products method, students need to have knowledge of place value (e.g., the 1 of 13 represents 10, not 1) and fluency with their multiplication number combinations.

**THE MOUNTAIN**

It is helpful to think about climbing a mountain as an analogy for establishing proficiency with foundational skills for a specific Common Core standard. A Common Core cluster lies at the top of the mountain. Standards fall below the cluster, such that when the foundational skills are integrated, they lead to mastery of the cluster. As students hike up the mountain and reach the series of base camps (i.e., foundational skills), they gain the skills and supplies necessary to reach the mountaintop with success.

Many teachers wonder how to justify working on foundational skills with their students with MD. We propose that working on foundational skills necessary to the cluster at the mountaintop is working on the Common Core, especially when the concepts for the mountaintop cluster are simultaneously addressed in general education (Lembke, Hampton, & Beyers, 2012). The triangle problem introduced earlier (i.e., calculating area of a triangle) is a standard in the cluster addressing “area, surface area, and volume” in the sixth-grade geometry domain. Instruction on foundational skills might include measuring sides of a triangle, multiplication and division concepts and procedures, and learning how to represent area with square units. While a teacher works on these foundational skills, the teacher also works on the concepts underlying the calculation of the area for triangles, rectangles, and circles. This way, students gain knowledge about and context for the standard, even as they have authentic opportunities to develop the foundational skills that will permit the students to reach the mountaintop. The instructional approach is slightly different than for students without MD, where the focus is more on the standard or cluster and less on foundational skills. When foundational skills are not established, students will continue to struggle with mathematics and not meet the Common Core standards.

In keeping with the mountain analogy, foundational skills can be thought of as base camps students must reach to pick up necessary supplies. By learning each of the foundational skills (or reaching the next camp), students are making progress in learning the cluster (or reaching the mountaintop). We are not suggesting teachers only focus on one skill at a time, but teachers should present students with an instructional program that works on several foundational skills simultaneously. Teachers must constantly monitor the progress of their students using formal mathematics progress-monitoring measures to determine if students are demonstrating adequate progress, when to move students to work on another foundational skill, and when to make other adjustments to the instructional program (Stecker, Fuchs, & Fuchs, 2008).

**Roscoe’s Mountain**

To help with this mountain analogy, we present Figure 3. This mountain is designed for a hypothetical third-grade student, Roscoe. Roscoe struggles with mathematics and has IEP goals related to mathematics. At the top of Roscoe’s mountain is a cluster within the Common Core domain of Numbers and Operations in Base Ten (NBT). The cluster requires students to “use place value understanding and properties of operations to perform multi-digit arithmetic” (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). As outlined by the Common Core, three standards are related to this cluster. These standards are highlighted in gray rectangles. We placed the standards on the mountain in terms of where they fall for Roscoe: Easier standards are at the bottom of the mountain;
more complex standards at the top. At the bottom of the mountain, Roscoe needs to learn how to use place value to round to the nearest 10 or 100. In the middle, Roscoe needs to learn how to fluently add and subtract within 100 (e.g., 34 + 29; 89 – 14). Toward the top of the mountain, Roscoe must be competent at multiplying 1-digit numbers by groups of 10 (e.g., 4 \times 30; 70 \times 9). These three foundational skills (place value with rounding, addition and subtraction within 100, and multiplication of 1-digit numbers by ten) all lead to conquering the cluster related to using place value to perform multi-digit arithmetic. While Roscoe’s teacher, Ms. Li, focuses on these foundational skills, she also works on the concepts of multi-digit addition, subtraction, and multiplication. She continues working on the concepts, using manipulatives, arrays, and real-life scenarios, so Roscoe can gradually understand how to use his burgeoning collection of foundational skills in the solving of multi-digit arithmetic and thereby avoid falling further behind his classmates on learning the third-grade cluster in Numbers and Operations in Base Ten.

Look at the foundational standard at the lowest point of the mountain: use place value to round to the nearest 10 or 100. For Roscoe to demonstrate success with rounding to the nearest 10 or 100, Roscoe must achieve several other foundational Common Core standards. Extending the counting sequence is probably the first, and therefore is placed lowest on the mountain. This standard comes from the NBT domain at first grade: competence with counting to 120. Students should be able to start at any number prior to 120 (e.g., 68) and count from that starting point. Before Roscoe can round, he needs to be able to count, and he needs to understand that counting does not stop at 10 or 100. Next on the mountain are the first- and second-grade standards from the NBT domain related to understanding place value. At first grade, this entails understanding that two-digit numbers represent amounts of tens and amounts of ones and that 10 is a bundle of ten ones. This standard also addresses comparing two two-digit numbers and determining greater than, less than, or equal to. At second grade, this foundational standard incorporates understanding that a three-digit number presents hundreds, tens, and ones, understanding that 100 is a bundle of ten tens, and being able to compare three-digit numbers. We placed the “counting to 120” standard next to and below the “understanding place value” standard, as students need to have established counting knowledge before working on what the counts represent. Even though these standards are categorized at first- and second-grade in the Common Core, it is important that Ms. Li be familiar with standards that Roscoe may not yet have acquired competency because these standards relate directly to the third-grade Common Core standards Roscoe presently faces.

We compiled the Common Core foundational standards that relate to the other two standards on Roscoe’s mountain in a similar fashion. For the middle-grade foundational standard on adding and subtracting within 100, Roscoe must add and subtract within 20. This addresses skill with number combinations. Roscoe also needs to represent and solve problems using addition and subtraction knowledge. This would build upon and practice his work with the number combinations. Roscoe would then work his way to using place value to add and subtract. Instruction in this area would focus on the algorithm (or multiple algorithms) Roscoe can use to solve single- and double-digit computation problems. All these skills, when addressed together, lead to Roscoe’s ability to fluently add and subtract within 100. For the foundational standard closest to the top of the mountain, multiply 1-digit whole numbers by 10, Roscoe needs to demonstrate proficiency on working with equal groups to understand multiplication and multiplying and dividing within 100. The latter is important to meet the top of the mountain because Roscoe will solve 4 \times 70 with greater ease and success if he knows that 4 \times 7 = 28.

**Instruction for Roscoe**

Consider Roscoe’s mountain again. Let us say Roscoe’s teacher, Ms. Li, had assessed Roscoe and determined he was able to use place value to round whole numbers. So, now Ms. Li looks at the middle-mountain foundational standard. She realizes that Roscoe is not fluent with adding and subtracting within 100. Ms. Li assesses Roscoe’s number combination knowledge. She finds that Roscoe does very well with addition number combinations but has difficulty with subtraction number combinations. She decides to provide instruction to Roscoe on the relationship between addition and subtraction and teach Roscoe an efficient counting strategy to solve unknown subtraction number combinations (e.g., Fuchs, Powell, et al., 2010). Ms. Li also incorporates a validated practice game on a computer tablet along with a timed paper-and-pencil review with immediate feedback from Ms. Li (e.g., Powell, Fuchs, Fuchs, Cirino, & Fletcher, 2009).

Ms. Li also decides that Roscoe can simultaneously work on representing and solving addition and subtraction problems. Ms. Li ensures that Roscoe knows the signs for addition and subtraction, that addition and subtraction problems can be presented horizontally and vertically, and that the equal sign or line means two sides of an equation represent the same amount. Ms. Li also provides word-problem instructional regression to Roscoe for additive problem types. She encourages him to read each word problem and circle the important information (i.e., labels and numbers) necessary for answering the word problem’s question. Ms. Li also teaches Roscoe how to write an addition or subtraction equation with missing information (e.g., 7 = 4 + __) to represent the structure of the word problem (e.g., Fuchs, Seethaler, et al., 2008).

As Ms. Li works on these foundational skills for Roscoe, she is helping him climb the mountain. As Roscoe is working on improved fluency in subtraction, Ms. Li begins work on double-digit addition and subtraction while continuing to address word problems. In this way, Ms. Li is working on the cluster at the top of the mountain even as she helps Roscoe achieve competence with the foundational skills involved in climbing the mountain. Roscoe participates in mathematics instruction with the rest of his class (where the focus is on the cluster), and his supplementary mathematics intervention focuses on the foundational skills and how the skills relate to the cluster. By spending time on the foundational skills, Roscoe eventually reaches the mountaintop. When appropriate,
Ms. Li will decide, via progress-monitoring assessments, when Roscoe is ready to start learning the relationship between repeated addition and multiplication so that Roscoe continues to make progress toward the mountain top. Once he begins working on the basics of multiplication, Ms. Li will continually reinforce and review the foundational skills important to multiplication of multi-digit numbers (i.e., place value and addition).

**GENERALIZING THE MOUNTAIN**

Figure 3 provides one example for how Common Core standards can be conceptualized for students with MD. The trail to the mountain top differs depending on the skills profile of each student with MD. Also, many students may have several “mountains” they work on concurrently. In Roscoe’s case, in addition to the mountain just discussed, he has the challenge of climbing a mountain focused on a geometry cluster and one focused on a measurement cluster. The IEP team decides, guided by Ms. Li’s judgment, how many mountains Roscoe can handle at any one time. Most students without MD receive mathematics intervention on several topics at the same time (i.e., place value and geometry and expressions). So, a multi-mountain (or mountain range) goal is typically not unreasonable. In fact, it may be the only way teachers can help a student work through a cumbersome mathematics curriculum during a school year. Additionally, as discussed earlier, many foundational skills appear time and time again, so foundational skills from one mountain may relate to foundational skills on another mountain.

When teachers design a mountain for a given student with MD, place the grade-appropriate Common Core cluster at the top of the mountain. The standards within that cluster are then located below the mountaintop, with the easiest standard at the bottom, working up through the more difficult foundational standards. Placing the cluster and gray rectangles (i.e., standards) is straightforward because the cluster and its standards come directly from the Common Core document. It is the foundational skills (in the white boxes) for each standard that are less clear-cut and more individualized. Teachers must look at other domains, clusters, and standards within the student’s grade level (and almost certainly below the student’s grade level) to develop a clear picture of the necessary foundational skills (as outlined by the Common Core) to meet the grade-level standards. As teachers place foundational skills on the mountain, they may realize that some students have already embraced certain foundational skills. It is necessary to place the foundational skills on the mountain as they relate to the standards and cluster so the teacher understands (and can check for student understanding on) the big picture related to the grade-level Common Core cluster.

**MOUNTAIN INSTRUCTION**

For designing instruction for the foundational skills and the cluster, teachers need to use validated or research-principled instruction. This is true for instruction provided within the general classroom as well as for any small-group or individualized instruction for students with MD (Lembke et al., 2012). When formulating decisions about which validated practices to use for instruction on foundational skills as they relate to a cluster or standard, researchers have determined several best practices for instruction for students with MD (Archer & Hughes, 2011; Doabler et al., 2011; Fuchs, Fuchs, Powell, et al., 2008). Validated instruction for students with MD should include some of the activities or at least follow some of the principles outlined in the introduction. These include explicit and systematic instruction, step-by-step modeling, heuristics, mnemonics, a combination of conceptual and procedural learning, visual representations, cumulative practice, and systematic, ongoing monitoring of student progress. Many of these best practices for students with MD align closely to the Common Core standards’ mathematical practices. To the greatest extent possible, teachers should use validated practices that incorporate some or all of the Common Core practices.

Several websites are helpful for starting the process of identifying validated practices, all of which use strict criteria to determine whether an intervention or program is effective and validated. First is the What Works Clearinghouse (ies.ed.gov/ncee/wwc/). Most programs on this site are related to general education. Second is the Best Evidence Encyclopedia (www.bestevidence.org/). This site includes a variety of general and interventions for students experiencing difficulty. Third is the National Center for Response to Intervention (www.rti4success.org/) which includes evaluations of intensive interventions. Although teachers have to read through each program and make the best decision for their students, these resources provide a good starting point.

**RECOMMENDATIONS FOR TEACHERS**

In this section, we provide with three recommendations for to help teachers conceptualize, justify, and provide appropriate mathematics intervention to help students with MD succeed with the mathematics Common Core. First, teachers should download and become familiar with the Common Core State Standards in mathematics. This document is lengthy, but well organized, and it outlines the expectations in mathematics for students from kindergarten through high school. Teachers should embrace the Common Core standards and understand that authentic changes need to be made to the current mathematics curriculum in their school and classroom (Beckmann, 2011). In becoming familiar with the standards, teachers will better understand the foundational skills necessary to address grade-level Common Core standards for their students with MD.

We also recommend that, for each student with MD, teachers select two or three Common Core clusters at the student’s grade level, to constitute that student’s mountain range of focus for a fixed time period (e.g., 1 month, 9 weeks, 1 semester). Teachers place each of these clusters at the top of a mountain diagram. (Please note, although we like the mountain analogy, teachers could also draw or type the cluster, related standards, and foundational skills on a blank piece of paper, a diagram of a ladder, etc.). Teachers then should
place the related standards for each cluster on the mountain from easiest (bottom of mountain) to the hardest (top of mountain). Teachers should assess students and determine which foundational skills and standards are necessary base camps for an individual.

Finally, teachers should select and develop instruction from validated practices that address the foundational skills and the grade-level Common Core cluster. Teachers should sequence the instruction logically so that easier foundational skills are taught before more difficult skills. Teachers need to create time within the school day to provide instruction for students (Larson, 2012). Teachers should continue providing instruction on the cluster as instruction is provided on the foundational skills. In doing this, students better understand the connections between foundational skills and the larger picture of mathematics. Teachers should remember the need to simultaneously provide conceptual work on the mountaintop Common Core standard while building strong foundational skills.

CONCERNS ABOUT THE COMMON CORE FOR STUDENTS WITH MD

At the same time that we introduce the mountain as a framework for working on the Common Core with struggling students, we are cautious what the Common Core will mean for students with MD. Our mountain analogy comes from prior work, before the era of the Common Core, for students with MD in which interventions were developed to address less challenging grade-level standards, concurrent with practice on foundational skills (e.g., Fuchs et al., 2005). Future work needs to investigate whether this approach is a viable option for teachers and students in the context of the Common Core.

While the Common Core presents a more cohesive set of standards than previous standards (Cobb & Jackson, 2011), some researchers and teachers question whether the Common Core will improve the quality of mathematics instruction in the U.S. (Lee, 2011; Porter et al., 2011). At the same time, some researchers question the lack of evidence supporting the Common Core standards (Tienken, 2011) and even wonder about the basis for this particular collection of standards. This concern is based, in part, on the fact that countries outperforming the United States on mathematics assessments focus more time learning procedures than concepts (Porter et al., 2011). These concerns may be particularly relevant to students with MD.

Several other questions arise with concern to the Common Core and students with MD. These include the following. First, should all students be required to learn the same content (Tienken, 2011)? For a country that embraces diversity, we often force students into a uniform curriculum instead of focusing on foundational skills (e.g., Fuchs et al., 2005). Future work needs to investigate whether this approach is a viable option for teachers and students in the context of the Common Core.

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Second, how will schools provide the additional, targeted instruction that students with MD need to succeed with Common Core requirements (National Mathematics Advisory Panel, 2008)? With the understanding that the Common Core standards require students to develop deeper understandings about mathematics, how will students who struggled with “old” standards succeed with more difficult standards and on more challenging assessments? Will schools give these students the time and instruction necessary to develop deep mathematical understanding or teachers feel the need to rush and cover the content (Russell, 2012)? How will schools that already have difficulty fitting necessary instruction into the school day provide the time and resources for additional support that students with MD require (Beckmann, 2011)? A third issue pertains to what Larson (2012) terms tracking. Larson (2012) argues that schools, both at the elementary and secondary levels, need to eliminate tracking that denies students the opportunity to take full advantage of the Common Core. But schools may find it necessary to use tracks—or for students with MD, the supplementary instruction required in RTI—to prepare students for Common Core assessments.

Fourth, how will schools deal with non-responders (i.e., students who do not demonstrate adequate progress after the implementation of evidence-based interventions; O’Connor & Klingner, 2010)? If the Common Core curriculum, even with additional supports and instruction built-in, does not improve the mathematics performance of students with MD, will schools discount these students and let them fall further and further behind? We hope the Common Core results in positive change to mathematics education in the U.S. We caution, however, that researchers and teachers need to investigate the effects of curricula and interventions aligned with the Common Core to determine the best practices for students who struggle with mathematics the most.

REFERENCES


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